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M.Sc. (Sem.- III) Examination October - 2025

Mathematics (Paper - PGMTH : 303) Calculus Of Variation

Time: 3 Hours] **[Total Marks: 70**

સૂચના : / Instructions

(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book

(2) Answer all questions

(3) Figures to the right indicate marks

(4) Follow usual notations and conventions

Seat No.:

Student's Signature

Q.l. Attempt Any Two: 14

1. Find theextremum ofthe functional $I[y(x)] = \int_0^1 [y^2 + 12xy]\, dx, y(0) = 0, y(l) = l.$
2. Define : Variation ofa functional $I[y(x)]$. Show that the variation of a functional $I[y(x)]$ is $\frac{\partial}{\partial a}\{I[y(x) + \alpha\delta y]\}$ *at* $\alpha = 0$, where y and δy are fixed and a is a parameter.
3. State and prove fundamental lemma ofcalculation.

Q.2. Attempt Any Two: 14

1. Obtain Euler's Equation for $I[y(x)] = \int_a^b F[x, y(x)]\, y'(x)]$ dx subject to the boundary condition $(a) = y_1, y(b) = y_2$.
2. Find the extremal of the functional $\int_{-l}^{l}\left[\frac{1}{2}\mu y''^2 + py\right] dx$ that satisfies the boundary condition $(-l) = 0$,$y'(-l) = 0$,$y(l) = 0$ *and* $y'(l) = 0$.
3. Find the curve with fixed boundary points such that its rotation about the axis of abscissae give rise to surface of revolution ofminimum surface area.

Q.3. Attempt Any Two: 14

1. Obtain Euler Ostrogradsky equation for the extremum of $\iint_D F[x, y, u, u_x, u_y]\, dxdy$ over the region D.
2. Find the extremals of functional
$I[y(x)], z(x)] = \int_0^{\frac{\pi}{2}} [y'^2 + z'^2 + 2yz]\, dx,$
with boundary conditions $y(0) = 0, y\left(\frac{\pi}{2}\right) = 1$, $z(0) = 0, y\left(\frac{\pi}{2}\right) = -1$.
3. If the integrand of the functional $I = \int_{t0}^{t_1} F[x(t), y(t), \dot{x}(t), \dot{y}(t)]\, dt$ does not contain t explicitly and it is a homogeneous function of first degree in $\dot{x}(t)$ and $\dot{y}(t)$ thenshow that I depends solely on the kind of curves $x(t)$ and y(t) noton parametrizatipn.

Q.4. Attempt Any Two: 14

1. Find the minimum distance between circle $x^2 + y^2 = 1$ and straight line $+ y = 4$.
2. State transversahty condition for the functional whose one boundary point movealongthe curve $\phi(x)$. Showthat the transversahty condition reduces to

orthogonality condition for the functional
$I[y(x)] = \int_{x1}^{x2} A(x, y)\ (1 + y'^2)^{\frac{1}{2}}\ dx$ where $A(x, y)$ does notvanish at movable boundary point x^2.

3. Test for extremum ofthe functional $I[y(x)] = \int_0^{x1} \frac{\sqrt{1+y'^2}}{y}\ dx$ $y(0) = 0$, and if the second boundary point (x_1, y_1) can move along the circumference of the circle $(x-9)^2 + y^2 = 9$.

Q.5. Attempt Any Two: **14**

1. Explainproper field and central field with example. Also define extremal field, embedding in a field of extremals and embedding in a central field.
2. Investigate for an extremum ofthe functional
$I[y(x)] = \int_0^1 \left(x + 2y + \frac{1}{2}y'^2\right) dx$, $y(0) = 0$, $y(1) = 0$.
3. Is the Jacobi condition fulfil for the extremal ofthe functional
$I[y(x)] = \int_0^a (y'^2 + y^2 + x^2)$ dxpasses through $A(0,0)$ and B$(a, 0)$?
